Calculus Warm Up #5-3

Find the area of the region bound by

$f(y) = -y^2 + 2y$ and $f(y) = -y$

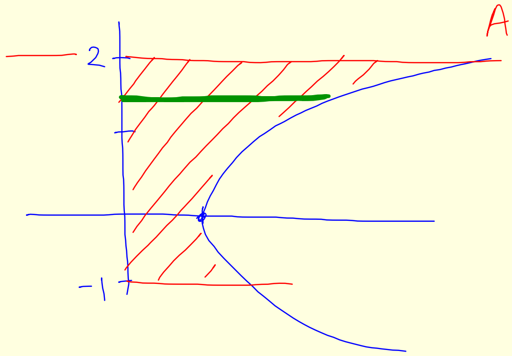
In Exercises 9–26, sketch the region bounded by the graphs of the given functions and find the area of the region.

21. $y = x^2 - 4x + 3$, $y = 3 + 4x - x^2$

23. $f(y) = y^2$, $g(y) = y + 2$

25. $f(y) = y^2 + 1$, $g(y) = 0$, $y = -1$, $y = 2$

$A = \int_{-1}^{2} (y^2+1)\,dy$



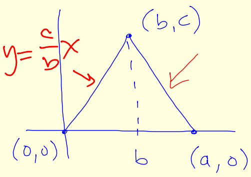
In Exercises 27 and 28, use a symbolic integration utility to graph the region bounded by the graphs of the functions and find the area of the region.

27. $f(x) = x^4$, $g(x) = 3x + 4$

29. Use integration to find the area of the triangle: (0, 0), (4, 0), (4, 4)

31. Use integration to find the area of the triangle: (0, 0), (a, 0), (b, c)

$y = \frac{c}{b}x$



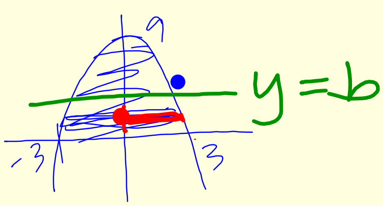
$y = \frac{c}{b-a}(x-a)$

$$\int_0^b \left(\frac{c}{b}x\right)dx + \int_b^a \left(\frac{c}{b-a}(x-a)\right)dx$$

$$\frac{c}{b}\int_0^b x\,dx + \frac{c}{b-a}\int_b^a (x-a)\,dx$$

33. Find b so y = b divides the region into two equal regions.

$y = 9 - x^2$, $y = 0$



$$\frac{1}{2}A = \int_0^3 (9 - x^2)\,dx$$

$$\frac{1}{2}A = 18$$

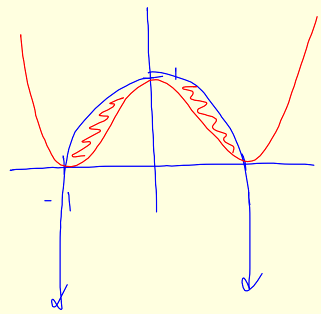
$$2\int_0^b (9-y)^{1/2}\,dy = 18$$

$x^2 = 9 - y$

$x = \sqrt{9-y}$

35. $y = x^4 - 2x^2 + 1$

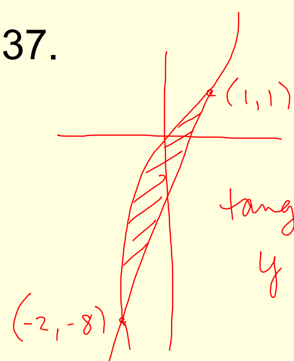
$y = 1 - x^2$



Symmetry

$2\int_0^1 (\text{top curve} - \text{bottom})\,dx$

37.



$f(x) = x^3$

tangent $m = f'(x) = 3x^2$

@ (1,1) $m = 3$

tangent line

$y - 1 = 3(x - 1)$

$y = 3x - 2$

intersection points: $x^3 = 3x - 2$

$x^3 - 3x - 2 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -3 & -2 \\ & & 1 & 1 & -2 \\ \hline & \underbrace{1 \quad 1 \quad -2} & & & 0 \end{array}$$

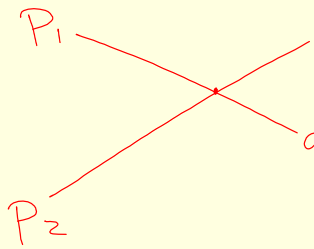
$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2, 1$

$$A = \int_{-2}^{1} (x^3 - 3x + 2)\,dx$$

45.



$s \Rightarrow p_2(x) = 0.125x$

$d \Rightarrow p_1(x) = 50 - 0.5x$

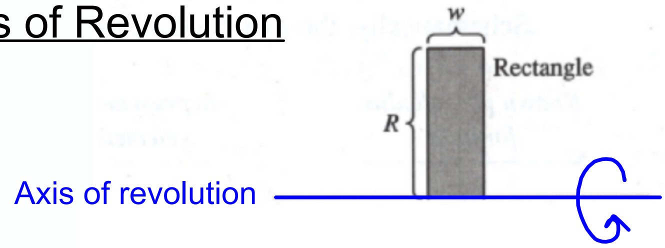
6.2 Solids of revolution

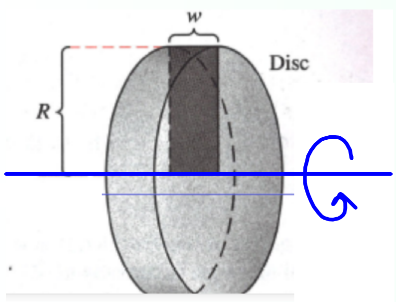
The disc method

The washer method

Solids with known cross sections

Solids of Revolution

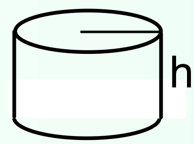




$$V = \pi R^2 w$$

R = height of rectangle
= Radius of the disc

Disc = Cylinder



$$(V = \pi r^2 h)$$

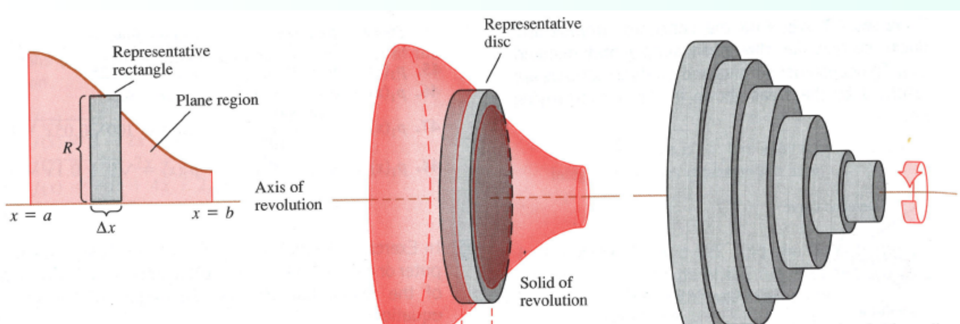


FIGURE 6.13

R = "radius as a function of x"

Volume of one disc

$$V = \pi R^2 \Delta x$$

$$V = \pi[R(x_i)]^2 \Delta x$$

Known precalculus formula		*Representative element*		*New integration formula*
volume of disc $V = \pi R^2 w$	$\longrightarrow$	$\Delta V = \pi[R(x_i)]^2 \Delta x$	$\longrightarrow$	solid of revolution $V = \pi \int_a^b [R(x)]^2\, dx$

To find the volume of a solid of revolution with the **disc method,** use one of the following as indicated in Figure 6.14.

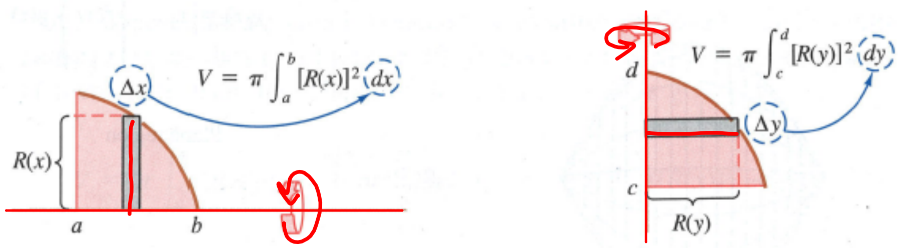
Horizontal axis of revolution

$$\text{volume} = V = \pi \int_a^b [R(x)]^2\, dx$$

Vertical axis of revolution

$$\text{volume} = V = \pi \int_c^d [R(y)]^2\, dy$$

REMARK In Figure 6.14 note that we can determine the variable of integration by placing a representative rectangle in the *plane* region "perpendicular" to the axis of revolution. If the width of the rectangle is Δx, we integrate with respect to x, and if the width of the rectangle is Δy, we integrate with respect to y.

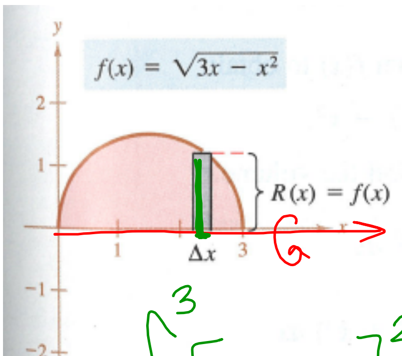


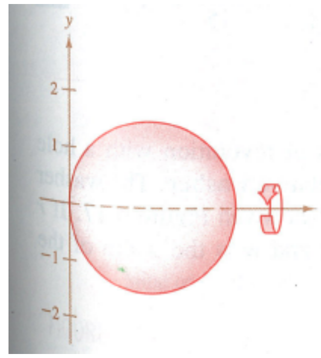
Horizontal axis of revolution

Vertical axis of revolution

You try:

Find the volume of the solid formed by revolving the region bounded by the graph of $f(x) = \sqrt{3x - x^2}$ and the x-axis $(0 \leq x \leq 3)$ about the x-axis.



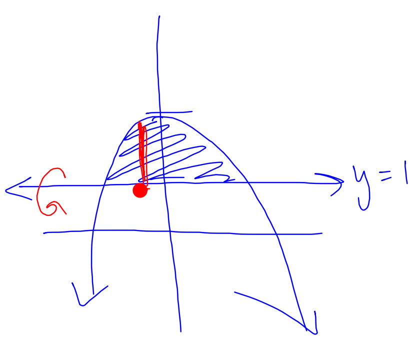


$$V = \pi \int_0^3 \left[\sqrt{3x - x^2}\right]^2 dx$$

$$= \pi \left[\frac{3x^2}{2} - \frac{x^3}{3}\right]_0^3$$

Find the volume of the solid formed by revolving the region bounded by

$f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$



$2 - x^2 = 1$

$x = \pm 1$

$$V = \pi \int_{-1}^{1} (2 - x^2 - 1)^2 dx$$

$$= \pi \int_{-1}^{1} (-x^2 + 1)^2 dx$$

The washer method

The disc method can be extended to cover solids of revolution with a hole by replacing the representative disc with a representative **washer.** The washer is formed by revolving a rectangle about an axis, shown in Figure 6.17. If r and R are the inner and outer radii of the washer and w is the width of the washer, then the volume is given by

$$\text{volume of washer} = \pi(R^2 - r^2)w.$$

Volume Cylinder
$V = \pi r^2 h$

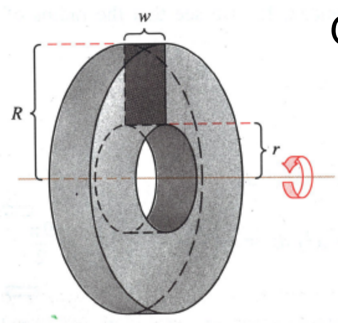


FIGURE 6.17

Outer cylinder - inner cylinder

$\pi R^2 w \quad - \quad \pi r^2 w$

$= \pi (R^2 - r^2) w$

on $[a,b] \rightarrow \quad V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2 \right) dx$

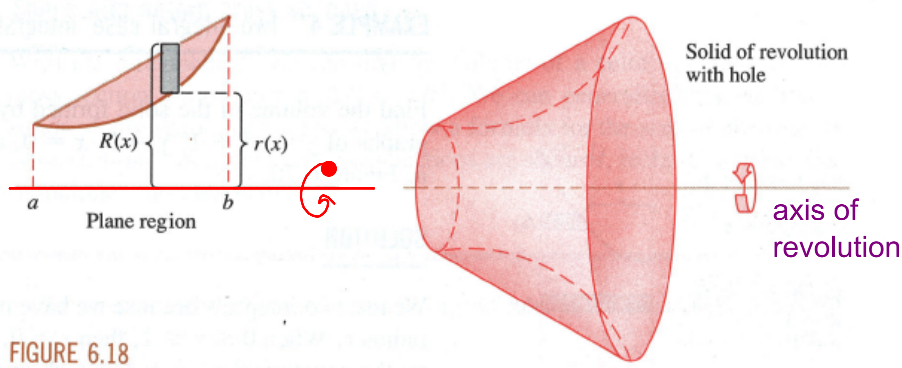


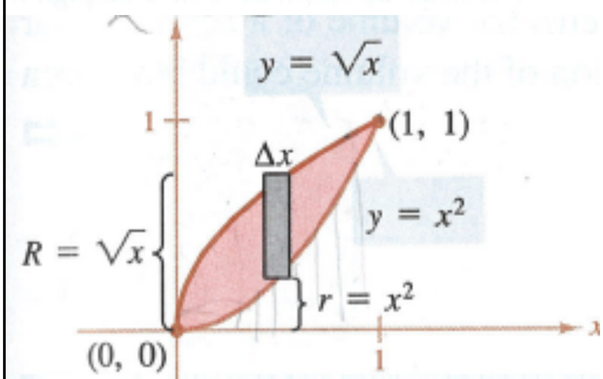
FIGURE 6.18

Now, suppose that a region is bounded by an **outer radius** $R(x)$ and an **inner radius** $r(x)$, as shown in Figure 6.18. If this region is revolved about its axis of revolution, then the volume of the resulting solid is given by

$$V = \pi \int_a^b [R(x)]^2\, dx - \pi \int_a^b [r(x)]^2\, dx = \pi \int_a^b ([R(x)]^2 - [r(x)]^2)\, dx.$$

You try:

Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ about the x-axis,



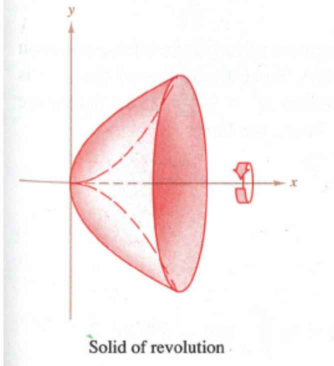


FIGURE 6.19

$$V = \pi \int_0^1 \left[(\sqrt{x})^2 - (x^2)^2 \right] dx$$

$$= \pi \int_0^1 (x - x^4)\, dx$$

HW: p. 310 # 1 - 13 odd